Radio competition and programming diversity

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Abstract

Broadcasting quotas of domestic contents are commonplace in developed countries. The main rationale for these quotas is to promote diversity by fostering domestic content. This justification ignores a possible trade-off between repetition and new program diffusion. When contents are imperfect substitutes and broadcasters seek to maximize audience (because of an ad cap for example), a broadcaster confronted to a quota will find optimal to compensate for the reduction of foreign programming by increasing the number of diffusions of substitutable domestic programs. Total broadcasting time being limited, this will force the broadcaster to abandon less popular types of programming, reducing program diversity.

Keywords: radio, broadcasting, cultural quotas, diversity, advertisement

JEL: L59, L82, Z10

1 Introduction

1.1 Motivation

The UNESCO Universal Declaration on Cultural Diversity of 2001 testifies that fostering diversity in broadcasting is a widespread concern for governments and media regulation agencies worldwide. Point 12 of the action plan explicitely mentions public service radio and television services as a tool of choice to that end. In practice, this concern with diversity reflects a fear of an hegemony of US cultural goods and services. Increasing diversity then means increasing the share of domestically produced goods. This goal is commonly pursued through a combination of mandatory quotas of domestic contents and operation of a public service broadcasting system. Rationales for promoting domestic contents abound, from merit goods arguments to the existence of externalities of production and consumption, but the most prominent argument is the will to preserve a lively local culture and the production of contents that may not exist if they faced the full competition of foreign-produced contents (see for example the motivation of François and van Ypersele (2002)).

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A quota, as well as dedicated time to national contents in the public service, can be expected to increase the share of domestic contents broadcast (whether these contents is actually listened to is another issue). It does not however provide guarantees about the composition of that domestic content. Additional time devoted to local artist may induce more entry from artist or the selection in playlists of contents that the broadcaster would not have other wise choosen. It may also induce local artists to produce contents akin to the foreign contents that the quota makes less available or induce programmers to select that type of production. If the latter occurs, the quota has the opposite of its intended effect since it provides incentives for national artists to become more like international ones, minimizing the local aspect in their production.

In this article, I do not explicitly consider how quotas affect artists' production and style decisions. I assume that both domestic and foreign production exists for any given type of content and concentrate on the decision of whether a given content will be broadcast or not. This means I take at face value two different arguments. Firstly, that there is indeed something special about local culture that makes it worthy of being protected, thatis, all other things being equal, programs with more domestic contents are socially desirable. Secondly, that diversity also provides social benefits and that more diverse programming is also socially desirable. In frameworks where the share of domestic contents is the choosen measure of diversity, the two propositions are obviously identical. However, when a other measure of diversity are taken into account, a trade-off may exist between the two policy objectives, as underlined by Ranaivoson (2007) with data on French radio broadcasting.[1]

The aim of this paper is to show when that trade-off can occur. The main finding is that when (i) domestic and foreign programs are not perfect substitutes for the consumers and (ii) broadcasters want to maximize audience (because of an exogenous objective or due to regulation constraints that limit advertisement), the broadcasters' optimal response to a quota is to increase the quantity of music devoted to the more popular segments and cutting the less popular types of content. Intuitively, a quota degrades the best mix of domestic and foreign content that the broadcaster can use, which means it must broadcast more of each content to reach any given level of utility among its consumers. Since broadcasting time is limited, this entails that less popular content do not fit anymore in the programming schedule. When broadcasters are allowed to choose freely both content and advertisement level, lowering advertisement levels is an alternative to an increase of the quantity of programming. Depending on consumers'

[1] Studying how diversity could be measured, Ranaivoson (2007) found that the share of French-language music did increase between 1997 and 2005, but shows that the number of French-speaking titles has actually decreased. No data on the number of titles exists for the years before 2003, but Ranaivoson notices that the rotation rate (the mean of the number of diffusions of a given song over a week) jumped from 3.3 in 1997 and 6.6 times in 2005. This increase is comparatively larger than French songs' gain in broadcasting share, which implies that the number of French titles must have decreased. As Ranaivoson notices, the actual jump in rotations may even be higher, since the panel of radios (which represents 95% of the audience) comprises generalist and public-service radios for which music does not represent the core of their programs. In addition, the use of a mean rotation rate may not accurately summarize the actual distribution of rotations since boradcasting appears to be very skewed: 2.4% of the titles (1575 out 64774 individual titles broadcast in 2008) represent 74,6% and the top 40 titles represent 44% of broadcasts, and more than 60% for radios targeted at a teenage audience.

concentration and aversion to ads, the optimal response to a quota can then increase the diversity of content broadcast.

This paper also shows that competition reduces the diversity of contents broadcast relative to what a monopoly would provide. While this broad result is a staple of the literature it hinges since Beebe (1977) on inefficient duplication of content between the competitors: different broadcasters provide the same (popular) content. In the framework of this paper, there is no overlap between the programs of the broadcasters and such duplication is ruled out. The diversity-reducing effect here stems from an inefficient duplication of content *within* each broadcaster's program in order to prevent the competitor from stealing the denser parts of the audience. This illustrates another channel by which competition can lower diversity.

The rest of this paper is organized as follows: the remaining of this section reviews the existing literature on this topic; section 2 presents the basic model; section 3 presents the main trade-off in the case of audience-maximizing broadcasters; section 4 extends the analysis to a profit-maximizing monopoly broadcaster and section 5 concludes.

1.2 Related literature

The contribution closest to the present paper is Richardson (2006) which explicitely focus on the question of broadcasting quotas, advertising and public broadcasting. In his framework, programming decisions take the form of a location decision on a Hotelling segment, the position of a radio reflecting the radio between domestic and foreign contents, and diversity is reflected by the distance between the two competing radios. He finds that a cultural quota as well as an advertising cap reduce differenciation between radios. This framework of spatial locatioon for representing choice is shared with several other important contributions to the question of the impact of advertising schemes on diversity, such as Gabszewicz et al. (2001), Gabszewicz et al. (2002), Gabszewicz et al. (2004), Dukes and Gal-Or (2003) and Anderson and Gabszewicz (2006). This literature provides insights on how pricing towards advertisers on the one hand and towards consumers on the other hand affect programming decisions. Those insights indicate that this market features significant two-sided dimensions. In addition, some papers, notably Vaglio (1995) and Bourreau (2003) use a similar framework to consider the interplay between differentiation and investment choices (labelled “quality”) under advertisement-supported or subscription-based revenue strategies.

This locational framework does not however allow to consider a measure of diversity *within* each type of content, that is the composition of the domestic and the composition of the foreign part of programming. To do so, I propose a setup closer to the seminal paper of Steiner (1952) where contents, independently of their origin, can be ranked by popularity. In Steiner's analysis, contents are taken for a discrete set and affected decreasing shares of the potential demand. Here, I consider a continuous product space, which allow me to assess the effect of the underlying concentration of tastes using standard distributions rather than *ad hoc* shares[2]. The difference between this framework and the usual locational model reflect a difference in what a relevant content is for economic purposes. In the locational model, any piece of news or music is substitutable to any other piece of content of the same kind and consumers care for kinds of contents. In this model, there exists an infinity of songs imperfectly substitutable to each other.

This departure from the locational model allows me to shed a new light on the question of duplication. In the line of Steiner (1952), Beebe (1977)and Spence and Owen (1977), locational models are concerned with the problem of a (possibly) inefficient duplication of contents between broadcasters (that is, different broadcasters selecting the same, popular but low-quality, content). In this model, the broadcasters choose the share of each type of content but also, within each type, the number of times a given content is broadcast. Thus, I can deal with duplication of content within each broadcaster's programming. While the locational framework provides a good depiction of television broadcasting (the prime example of that literature) where long-term programming grids are the rule, duplication of content is widespread is radio broadcasting (as well as in the increasing number of web-based streaming services), where the selection of contents changes each week and many pieces of content (*e.g.* songs) are broadcast several times each day.

The main extension of this paper bears also some relation to the literature exploring the impact of advertisement on content selection. A large share of the papers in that branch (for example Bourreau (2003), Gabszewicz et al. (2004) and the related contributions) have explored the consequences of the choice between advertisement-based and subscription-based business models for commercial medias. In this model, I restrict the analysis to the advertisement-based model which is prevalent in the music radios sector and, to this date, also applies to the "free" (*i.e.* non-premium) accounts on streaming music websites. While this means I do not consider explicitly the two-sided aspect of the broadcasting market my model a motive of that kind is nevertheless present: disutility from hearing ads works as a fee payed by consumers (as in Anderson and Gabszewicz (2006) where this disutility is part of the "real price") and, through a total time constraint, this disutility impact total payments received from advertisers, even though I assume that the per-listener price charged to advertisers is constant.

2 The Model

We consider media companies that want to maximize audience. To simplify the exposition of the model, we will use the setup of music radio stations : media companies (platforms) are radios, content are music titles and content providers are record labels.

[2]This move from the a discrete to a continuous product space is common in the literature stemming from Dixit and Stiglitz (1977). It is here further vindicated by the high number of product classes used in the empirical analysis of the market, *e. g.* Rogers and Woodbury (1996), Berry and Waldfogel (2001) and Sweeting (2010).

2.1 Content and consumers

2.1.1 Music

We represent music by a continuum of music "genres". Genre, or alternatively class of titles, are ranked along $[0, +\infty[$ by a decreasing popularity index π. The closer π is to 0, the greater the number of consumers interested by this genre. Titles are provided by record labels, who fall into two categories : D (domestic) and F (foreign) music. In each genre, there is one D title and one F title. A *musical program* is then given by $\Pi \subset \mathbf{R}_+$ the set of genres broadcast by a given radio, $m_D(.)$ the number of broadcasts of the D title for each $\pi \in \Pi$ and $m_F(.)$ the same for F titles.

2.1.2 Consumers

There is a unit mass of consumers, and each consumer is interested by one and only one genre of music. Hence, decreasing popularity means that consumers can also be indexed by π (the genre they like) over $[0, +\infty[$ following a decreasing distribution function f, of cumulative F. Each consumer is hence a kind of "buff" of her favourite music, and music genres with a lower π have more fans than those with an higher π.

In order to model a trade-off between foreign and domestic music, I assume that foreign and domestic music consumption feature some sort of complementarity. More specifically, I assume that a consumer listening to a couple m_D, m_F of her favourite genre enjoys utility $u(m_D, m_F)$, with:

$$\begin{aligned}
&\forall i, j \in \{1, 2\}, i \neq j, \forall (m_D, m_F) \in \vec{R}^2_+,\\
&u'_i(m_D, m_F) > 0,\\
&u''_{ii}(m_D, m_F) < 0\\
&u''_{ij}(m_D, m_F) > u''_{ii}(m_D, m_F),\\
&u''_{ij}(m_D, m_F) > u''_{jj}(m_D, m_F)
\end{aligned}$$

where u'_i denotes the derivative of u with respect to its i-th argument.[3] Utility is thus increasing and concave in each of its arguments. Although standard in general literature, this assumption is not completely straightforward for cultural goods. Some of them feature increasing marginal returns of consumption over time. Since this model is static in nature however, diminishing marginal returns seems a sound assumption for consumption at a given point of time.

The assumption of the relative values of second- and cross-derivatives reflects the idea that as the level of one type of music increases, the marginal utility of hearing more music of the

[3] For example, the standard Dixit-Sitglity utility with two goods $u(m_D, m_F) = \left(m_D^\rho + m_F^\rho\right)^{\frac{1}{\rho}}, 0 < \rho < 1$ features these properties.

same type decreases more quickly than hearing more music of the other type. Notice that cross-derivatives need not be positive. This reflects a form of taste for diversity since, above a threshold, consumers get tired of hearing one type of music sooner than of hearing a more balanced mix of domestic and foreign music. This assumption is key to the results of this paper, since it lays the foundations of a trade-off between foreign and domestic music.

It should be noted that I assume that the function $u(.,.)$ is the same for all listeners, that is listeners are heterogeneous only with respect to the genre of music they like, and not with respect to the utility they derive from the consumption of a given level and mix of music. Since this model deals with the question of quotas, it is meant to be applicable to music genres covered by such quotas (e.g. pop-rock, R&B, rap) where the linguistic issue matters, and where tastes can arguably be considered as similar across genres.

While listening to the radio entails no direct cost, it means foregone opportunities to do other things: listening to recorded music, watching a film, and so on. Thus, I assume the opportunity cost of listening to the radio is $\gamma \in \vec{R}_+$, constant across consumers. I assume that a potential listener who is indifferent between listening and not listening will listen to the radio. A radio must hence guarantee its listeners at least a utility level equal to γ.

2.2 Media companies

2.3 Media companies

Radios provide music to consumers and sell audience to advertisers. Since the focus of this paper is on the impact of the quantity of advertising on programming choices, I assume that the price per listener charged to advertisers is constant (and equal to one). A radio thus chooses the genres it broadcasts and a level of broadcasts of domestic and foreign music for each genre selected. This means a radio chooses a set $P \subset \mathbf{R}_+$ of positive measure and for each $x \in P$ the programming levels $m_D(x), m_F(x)$. A programming strategy thus sums up to the choice of $A, m_D(.), m_F(.)$. The program is submitted to a total time constraint T, with one broadcast of any title taking one unit of time. For the sake of simplicity, I assume that the time devoted to ads is negligible and only consider the time devoted to music when assessing the effect of the constraint.

Let W_i be the set of consumers listening to a given radio R_i. Since consumers of a given type π are identical, either all of then listen to the same radio, or none of them listens to any radio or, when several radios provides the same level of utility, they split evenly between those radios. For the sake of clarity, let us assume for the moment that the third possibility is ruled out. Let then I be the set of radios, and let $u_{\pi,i}$ denote the utility of a consumer of type π listening to radio $i \in I$. Then :

$$W_i = \left\{ \pi \middle| \forall j \in I, j \neq i, u_{\pi,i} \geqslant \max_{I \setminus i} \{0, u_{\pi,j}\} \right\}$$

Hence, the total audience of R_i is given by the total number of consumers in W_i, that is:

$$\sigma(W_i) = \int_{W_i} f(u)\mathrm{d}u$$

which is well-defined since f is integrable and W_i is a subset of f's support. Hence, the profit-maximizing program of a radio is given by:

$$\max_{(P,m_D(.),m_F(.),a)} \left\{ a \int_{W_i} f(u)\mathrm{d}u \right\} \tag{1}$$

$$\text{s.t. } W_i = \left\{ \pi \,\middle|\, \forall j \in I, j \neq i, u_{\pi,i} \geqslant \max_{I \backslash i} \{0, u_{\pi,j}\} \right\} \tag{2}$$

$$\int_0^{+\infty} (m_{iD}(u) + m_{iF}(u))\,\mathrm{d}u \leqslant T \tag{3}$$

In this model, there is no marginal cost for a radio to broadcast more music, or a new genre of music. This stems from the cost structure of the industry and the type of contracts linking media companies and music producers. On the radio side, programming decisions are made weekly or monthly (see Caves (2002)), while operating costs (studio time, salaries, etc) are sunk for the whole year or more. On the production side, broadcasters do not contract directly with music production firms. They acquire broadcasting rights from copyright collectives, such as the RIAA, in the form of blanket licences that cover a wide spectrum of artists and genre. Hence, the expense for broadcasting rights can be also considered as sunk when programming decisions are made (see Connoly and Krueger (2006) for an overview). I thus consider that radio costs are summed up by a sunk cost K normalized to zero.

2.4 Diversity

As stated above, the main focus of this paper is how competition and regulation affect media companies' programming. In this setup, the main measure of program diversity is the measure of the musical genres that eventually get broadcast by one radio or another. Usually[4], the measure of diversity used is the share of domestic content broadcast at the equilibrium in a setup where consumers have different preferences over the ideal share. Indeed, the share of domestic content was the instrument chosen by the regulators when they set broadcasting quotas. The point of this article being to show that this is a poor, and indeed potentially confusing, approach to diversity. I use a simple metric of the usual sense of diversity in programming: programs appealing to different listeners. This, in turn, is only part of a proper characterization of diversity: the number of songs broadcast in each genre could also be a relevant metric of diversity, which is fundamentally a multidimensional issue (see Benhamou and Peltier (2007) on that topic). For the

[4]See Richardson (2006), Richardson (2004), Doyle (1998), Anderson and Coate (2005), Anderson and Gabszewicz (2006).

purpose of this paper however, it is enough to show how using one dimension of diversity (market share) as a policy objective can negatively impact another dimension, namely the measure of genres broadcast. In the rest of this paper, "diversity" will thus refer only to that latter metric for diversity.

3 Audience-maximizing radios

In this section we assume that radios maximize audience under the constraint that they must provide listeners a utility at least equal to $\gamma > 0$. This is equivalent to assuming that they are subject to an ad cap $\overline{a}$ such that $\overline{a}^r = \gamma$ and that this ad cap is binding in all the relevant cases. The consequences of the level of $\overline{a}$ on diversity will be considered in section 4.

3.1 Monopoly radio

In this subsection, we assume there is only one radio, which holds a monopoly position in the broadcasting market. This subsection presents the core intuitions on the optimal programming schemes and the response to a quota of domestic music for an audience-maximizing radio. Proposition 3.1 states that a monopoly faced with a quota broadcasts a lower measure of genres, cutting programming of less popular genres.

3.1.1 Monopoly programming

Let us first consider the optimal programming when there is no quota constraint:

$$\begin{aligned} &\max_{(A, m_F(.), m_D(.))} \{F(b) - F(a)\} \\ &\text{subject to: } \int_A [m_D(\pi) + m_F(\pi)]\,\mathrm{d}\pi \leqslant T \\ &\forall \pi \in A, u(m_D(\pi), m_F(\pi)) \geqslant \gamma \end{aligned} \tag{4}$$

The radio aims to attract as many listeners as possible within its time constraint T. Since there is no competition other than the outside option, which provides a utility level γ, the optimal programming strategy entails providing exactly utility γ to each listener with as little programming as possible.

Lemma 3.1. *A monopoly radio broadcasts on* $[0, \pi^*]$ *a constant level* m^* *of music, with:*

$$m^* = \min_{m = m_D + m_F} \{m | u(m_D, m_F) = \gamma\}$$
$$\pi^* = \frac{T}{m^*}$$

Proof. With no other constraint than a total broadcasting time, a monopoly radio locates on the denser part of the demand and just saturates listeners' participation constraint. This is done by choosing m^*:

$$m^* = \min_{m=m_D+m_F} \{m | u(m_D, m_F) = \gamma\}$$

Each listener is thus served m^*, which allows the monopoly to capture all consumers between 0 and $T/m^* = \pi^*$. $\square$

While that result is almost immediate, it is useful for what follows to consider how m^* is composed in terms of domestic and foreign music.

Let us consider a radio willing to provide a level of utility γ to some listener and starting its programming from scratch. The radio will first program whichever type of music (domestic or foreign) provides more utility to the consumer. Assume this is foreign music. Since $u''_{22} < u''_{12}$, marginal utility of foreign music u'_2 decreases more rapidly than marginal utility of domestic music u'_1 as the radio adds more foreign music. There thus exists a level of foreign music $\tilde{m}_F$ where $u'_1(0, \tilde{m}_F) > u'_2(0, \tilde{m}_F)$, where the consumer would rather hear some domestic music rather than more foreign music. If $u(0, \tilde{m}_F) < \gamma$, our radio will optimally start broadcasting some domestic music alongside the foreign songs. In turn, this makes the marginal utility of domestic music decrease more quickly than the marginal utility of foreign music, and there exists a point where it is optimal for the radio to cease to add domestic music and resume adding foreign songs. This process continues until the utility level γ is reached.

In what follows, I assume that u and γ are such that equilibrium programming is not degenerate, that is at equilibrium, the radio broadcasts both foreign and domestic music.

3.1.2 Monopoly quota programming

Assume now that a regulator imposes a regulation such that a share $\alpha \in [0, 1]$ of the broadcasting time must be devoted to domestic content (type D titles). For things to be interesting, the quota constraint needs to be relevant, that is $\pi^* m^*_D < \alpha T$. The radio must now maximize its audience under an additional constraint:

$$\max_{(A, m_F(.), m_D(.))} \left\{ \int_A f(x) \mathrm{d}x \right\} \tag{5}$$

$$\text{subject to: } \int_A [m_D(\pi) + m_F(\pi)] \, \mathrm{d}\pi \leqslant T \tag{6}$$

$$\int_A [m_D(\pi)] \, \mathrm{d}\pi \geqslant \alpha T \tag{7}$$

$$\forall \pi \in A, u(m_D(\pi), m_F(\pi)) \geqslant \gamma \tag{8}$$

The mechanic of the optimal quota programming remains akin to that without the quota. Lemma 3.2 states that formally.

Lemma 3.2. *With a quota, the optimal programming features a constant level of domestic and foreign music across genres covered. These levels verify:*

$$m_F^{**} = \frac{1-\alpha}{\alpha} m_D^{**} \tag{9}$$

$$u\left(m_D^{**}, \frac{1-\alpha}{\alpha} m_D^{**}\right) = \gamma \tag{10}$$

Proof. See Appendix A.1.1 □

The intuition of the proof is as follows. Let us assume that the radio wants to cover a given segment $[0, \pi_\alpha]$. Because the cost in terms of time of capturing an interval $[\pi, \pi + \varepsilon]$ does not depend on π, $m_D(.)$ and $m_F(.)$ will be constant over the segment, and we can reason point-wise. For a given π the radio provides the most favoured music type until the marginal utility of listening to the other type becomes larger. For any $\pi < \pi^{**}$, the quota limit will bite at some point, and the radio will make up the rest of programming only with domestic music. This naturally leads to proposition 3.1.

Proposition 3.1. *The interval of genres $[0, \pi^{**}]$ broadcast by a monopoly radio under a quota constraint is smaller than $[0, \pi^*]$. The measure π^{**} of this interval is decreasing with α.*

Proof. Proposition 3.1 states that $\pi^{**} < \pi^*$ and that $\frac{\partial \pi^{**}}{\partial \alpha} < 0$.

Let us start by the first relation. Since π^{**} is the result of the same maximization program as π^* with an additional (binding) constraint, it is immediate that $\pi^{**} \leqslant \pi^*$, with a strict inequality when the constraint is strictly binding. In the detail, lemma 3.2 states that the quota compels the radio to broadcast a suboptimal mix of domestic and foreign music (compared with the case when the quota constraint does not apply). In order to provide the same level of utility γ to its listeners, the radio must increase the time devoted to each genre above its level without a quota. Total broadcasting time being limited, the radio must then reduce the scope of genres it broadcasts.

Faced with a tightening of the constraint, that is $\alpha' > \alpha$, the radio's best response is to keep the denser genres, f being decreasing. Since (8) is saturated, it cannot reduce neither m_D nor m_F. Thus, it must reduce the lower bound of the interval of genre it serves, that is $\pi' < \pi^{**}$, or equivalently $\frac{\partial \pi^{**}}{\partial \alpha} < 0$. In technical terms, since the constraint is binding, the Lagrange multiplier associated with constraint (7) is positive. □

The intuition behind proposition 3.1 is simply that the quota forces the radio to program domestic music where foreign music would provide more utility to listeners. In order to reach the cutoff utility γ the radio thus needs to provide more programming to each genre it covers. Under a constant total time constraint T, this implies to cut programming for the less popular genres in order to increase programming for the more popular ones. This means the less popular genres get excluded, which lowers the diversity of music broadcast *domestic as well as foreign.* If we take the view that there are only a limited number of worthy songs in each genre, the rotation rate (the number of times a song is broadcast over a given period) of all domestic songs will increase but the number of different domestic songs that are broadcast at equilibrium is lower under a quota. The key assumptions for these results are that listeners care only for a limited number of genres and that domestic and foreign music are imperfect substitutes of each other in the sense that there exists some form of complementarity between the two types of music in the utility function.

3.2 Radio competition

In this subsection I consider two radios R_1 and R_2 competing for audience. In the light of the monopoly case and in order to make competition more tractable, I restrict *a priori* the radios' programming strategies be be constant levels on compact sets $[a, b] \subset \mathbb{R}_+$. On those sets,

$$\begin{aligned} \forall \pi \in [a,b], \quad & m_D(\pi) = m_D \\ & m_F(\pi) = m_F \\ \forall \pi \notin [a,b], \quad & m_D(\pi) = m_F(\pi) = 0 \end{aligned}$$

that is, the radios broadcast a constant level of domestic and foreign music respectively.

Radio i's strategy can then be described by the four choice parameters $(m_F^i, m_D^i) \in \mathbb{R}_+^2$, its programming for any given genre covered and (b^i, c^i) defining the segment $[b^i, c^i]$ of genres covered by this radio. The four parameters are bound together by the total time constraint and a quota if one exists. The best-response program of radio i is thus:

$$\begin{aligned} \max_{\{a^i, b^i, m_D^i, m_F^i\}} & \left\{ \int_{a_i}^{b_i} f(u) \mathbf{1}_{\left[u(m_D^i(u), m_F^i(u)) \geqslant \max\{\gamma, u_{j \in I \setminus i}(m_D^j, m_F^j)\}\right]} \mathrm{d}u \right\} \\ \text{s. t. } & (b^i - a^i)(m_D^i + m_F^i) \leqslant T \end{aligned}$$

Even with the restriction above, the simultaneous-move competition game admits no pure strategies equilibrium (see A.1.2 for a proof).[5] I therefore adopt a framework of sequential competition one of the radios acting as a Stackelberg leader.

3.2.1 Sequential competition

From this subsection on, I will assume that Radio 1 is an incumbent. It chooses its programming before Radio 2 (the entrant) does. Radio 1 thus acts as a Stackelberg-leader in the competition

[5]The author has been directed to the idea that this game is a version of the Colonel Blotto game with an infinite (continuum) number of battlefields — each music genre. Robertson (2006) characterizes solutions of this game in the case of a discrete number of battlefields, but, to the best of our knowledge, there exists no result concerning this variant of the game.

game.

This competition game features a winner-takes-all kind of dynamic: if the two radios broadcast the same genre, only the one with the higher level of programming for that genre gets all the relevant audience. As a result, equilibrium strategies never include an overlap between the two radios' programs (this intuition is formally proved in A.1.3). taking that into account, two strategies are possible for the incumbent radio (radio 1):

- **Popular incumbent** : Radio 1 can settle on the most popular genres, broadcasting all genres between 0 and some π_1^l, radio 2 catering to the $\left[\pi_1^l, \pi_1^l + \frac{T}{m^*}\right]$ segment.
- **Niche incumbent** Radio 1 can also settle further down the popularity scale on some $\left[a_r^1, b_r^1\right]$ segment, letting Radio 2 broadcast on $\left[0, a_r^1\right]$. choosing the bounds so that radio 2 prefers to broadcast on $\left[0, a_r^1\right]$ rather than competing for 1's leftmost listeners.

The following proposition states that the first strategy always dominates the second.

Proposition 3.2. *Radio* 1 *always settle on a* $[0, \pi_1^*]$ *segment and Radio* 2 *on* $\left[\pi_1^*, \pi_1^* + \frac{T}{m^*}\right]$*, with* π_1^* *such that*

$$F\left(\pi_1^*\right) = F\left(\pi_1^* + \frac{T}{m^*}\right) - F\left(\pi_1^*\right)$$

and m^* *the optimal level of programming derived in the monopoly case.*

Proof. See A.1.3 for a full proof. The main features are given below. □

The idea of the proof hinges on two features of that game. Firstly, by locating on the same segment as Radio 1 (minus some ε), Radio 2 can always do as well as Radio 1 in terms of audience. Due to that second-mover advantage, Radio 1 will ensure that at equilibrium, Radio 2 enjoys an audience at least equal as its own. Secondly, allowing an overlap between the program of the two radios is always (sometimes weakly) dominated for both radios. Thus, at equilibrium, the supports of the two programming will be disjoint.

Popular incumbent Assume first that Radio 1 follows a "Popular incumbent" strategy and settles on a $[0, \pi_1]$ segment. Radio 2 best response is either to compete for audience or accommodate and settle on some $[a_2, b_2]$ segment with $a_2 \geqslant \pi_1$.

If Radio 2 chooses to accommodate, its program is:

$$\begin{aligned}
&\max(a_2, b_2, m_D^2, m_F^2)\left\{F(b_2 + a_2) - F(a_2)\right\} \\
&\text{s.t.}(a_2 - b_2)(m_D^2 + m_F^2) \leqslant T \\
&\quad \forall u \in [a_2, b_2],]u(m_D^2(u), m_F^2(u)) \geqslant \gamma \\
&\quad a_2 \geqslant \pi_1
\end{aligned}$$

This program is identical to that of a monopoly radio constrained by $a_2 \geqslant \pi_1$. Radio 2 thus optimally behaves as a monopolist, and its location is of the form $\left[a_2, a_2 + \frac{T}{m^*}\right]$, where m^* the optimal level of programming derived in the monopoly case. Since the density of its audience $F\left(a_2 + \frac{T}{m^*}\right) - F(a_2)$ is strictly decreasing in a_2, it optimally locates on $\left[\pi_1, \pi_1 + \frac{T}{m^*}\right]$.

If Radio 2 chooses to compete for audience, it must serve its listeners a level of music strictly higher than the $\frac{T}{\pi_1}$ that Radio 1 provides. It will thus cover a segment $[a_2, a_2 + \pi_1]$. Once again, the audience on that segment is decreasing with a_2, so Radio 2 will compete head-to-head with Radio 1 and settle on $[0, \pi_1 - \varepsilon]$.

Since competition leaves Radio 1 with an infinitesimal audience, it must ensure that Radio 2 will prefer accommodating. It must thus choose π_1 such that the audience on $\left[\pi_1, \pi_1 + \frac{T}{m^*}\right]$ is equal to its own audience, $F(\pi_1)$. Hence the characterization of the optimal cut-off π_1^*:

$$F(\pi_1^*) = F\left(\pi_1^* + \frac{T}{m^*}\right) - F(\pi_1^*) \tag{11}$$

The proof in the appendix show that this cutoff is unique if f is strictly decreasing.

Niche incumbent If Radio 1 follows a "Niche incumbent" strategy and settles on a $[b_r^1, c_1^d]$ segment, the setup is basically the same. Radio 2 either accommodates, broadcasting on $[0, a_r^1]$ or competes, which means locating on $\left[0, \frac{T}{b_r^1 - a_r^1}\right]$. Radio 1 always prefer to avoid competition. Assume that $b_r^2 = \frac{T}{b_r^1 - a_r^1} > a_r^1$. Then, there exists an overlap between the two programs, and Radio 1 would have been better off locating on $[b_r^2, b_r^1 + (a_r^1 - b_r^2)]$: it would have attracted more audience, and the reaction of Radio 2 would have been identical. Radio 1 program is thus:

$$\begin{aligned}
&\max(a_r^1, b_r^1, m_r^1, m_F^1) \left\{F(b_2 + a_2) - F(a_2)\right\} \\
&\text{s.t.}(a_r^1 - b_r^1)(m_D^1 + m_F^1) \leqslant T \\
&\quad \forall u \in [a_r^1, b_r^1], u(m_D^1(u), m_F^1(u)) \geqslant \gamma \\
&\quad F\left(a_r^1\right) \geqslant F\left(\frac{T}{b_r^1 - a_r^1}\right)
\end{aligned}$$

This set is non-empty, and admits a smallest elements in terms of a_r^1. Let a_r^* denote that element and b_r^* the associated bound.

Once the payoffs of the two strategies are spelt out, the intuition behind the result is that Radio 1 must ensure that Radio 2 gets an audience at least as large as its own (with that being an equality in the popular incumbent case). If it prefers strictly the niche strategy, this means that it attracts more audience that way, and consequently that Radio 2 also attracts more audience than what Radio 1 would with a popular incumbent. This implies that a^* is larger than π_1^*. However, this means that the audience made on the $[a_r^*, b_r^*]$ is lower than what could be made on $\left[\pi_1, \pi_1 + \frac{T}{m^*}\right]$, which is equal to the audience on $[0, \pi_1^*]$ that Radio 1 would make if it choose the popular incumbent strategy, a contradiction.

An interesting feature of that equilibrium is that listeners of the incumbent enjoy a utility that is strictly greater than γ while listeners of the other radio get a utility just equal to γ. This result is consistent with an increase of the rotation rate following entry by a new radio.

The outcome of the competition game readily compares with the monopoly outcome. In order to have a proper benchmark, "monopoly" will here refer to a single radio with no competitor and endowed with $2T$ broadcasting time.

Proposition 3.3. *At the competitive equilibrium, the diversity of genres broadcast is lower than with a monopoly.*

Proof. This result is immediate. Since the monopoly exactly saturates listeners' participation constraint, any other programming strategy that is compatible with consumer participation over all its genres will entail less diversity. □

The result of proposition 3.3 is in line with some of the common arguments about program diversity (again, see Wurff (2005) and citations therein). The competitive equilibrium in this model features both a reduction of diversity and an increase in the broadcasting of the most popular genres. If one wishes to translate "most popular" by "low brow" and "less popular" by "high brow", this mirrors the argument of a dumbing down of programming compared to what a monopoly would do. It can also be noted that both effects are more pronounced when f decreases steeply.

3.2.2 Competition with quotas

Assume now that both radios are held to a quota of a share α of music of type D. Subsection 3.1.2 showed how saturating the quota uniformly across genres is the more efficient way to allocate D and F music. In subsection 3.2.1, I explained why competition between radios in my framework led to a concentration of programming and increased the surplus to listener of the most popular titles. Competition with quotas will combine those two insights.

Lemma 3.3. *At the competitive equilibrium, Radio* 1 *always settle on a* $[0, \pi_1^{**}]$ *segment and Radio* 2 *on* $\left[\pi_1^{**}, \pi_1^{**} + \frac{T}{m^{**}(\alpha)}\right]$*, with* π_1^{**} *such that*

$$F\left(\pi_1^{**}\right) = F\left(\pi_1^{**} + \frac{T}{m^{**}(\alpha)}\right) - F\left(\pi_1^{**}\right)$$

where $m^{**}(\alpha)$ *is the optimal level of programming derived in the monopoly case with a quota.*

Proof. The principle of this proof is to show that the presence of quota does not affect the logic of the proof 3.2, that is neither radio can make a strategical use of the existence of a quota.

To see that quotas are not used strategically at equilibrium, assume first that one of the radios serves a mix such that on an interval of non-zero measure, $m_F \neq \frac{1-\alpha}{\alpha} m_D$, that is it chooses to deviate from the optimal reaction to a quota derived in the monopoly case. From the proof of lemma 3.2, we know that that radio could offer the same level of surplus to its listeners while using less programming type. Therefore, a mix of this kind if not optimal. At the equilibrium with quotas, both radio thus serve a mix of domestic and foreign music such that $m_F = \frac{1-\alpha}{\alpha} m_D$.

Now, consider the proof of proposition 3.3. Let $m^{**} = m_D^{**} + m_F^{**}$ denote the quantity of music of any genre played at the optimal mix under a given quota. The proof of equilibrium selection is then identical, replacing m^* by m^{**}.

Thus, the equilibrium with quotas features both radios offering a mix such that $m_F = \frac{1-\alpha}{\alpha} m_D$ and Radio 1 following a "popular incumbent" strategy. □

The equilibrium of competition with quotas is thus similar to competition without quotas. Each radio responds to the quota in the same way a monopoly does, that is by cutting programming on less popular titles in order to compensate listeners of more popular titles for the lower utility of a sub-optimal mix. It is therefore natural that the diversity-reducing effect of a quota that we saw with a monopoly radio carries out to the competion case.

Proposition 3.4. *At the competitive equilibrium with a binding quota, the measure of genres broadcast is lower than without a quota.*

Proof. From the lemma 3.3, we know that the equilibrium with quotas is of the form $[0, \pi_1], [\pi_1, \pi_1 + \frac{T}{m^{**}}]$ with the incumbent (radio 1) located on the first segment and the entrant (radio 2) on the second one. Remember that the equilibrium condition is that audiences are equal in both segments, that is $F(\pi_1) = F(\pi_1 + \frac{T}{m^{**}}) - F(\pi_1)$.

Let us first consider the effect of a quota on the entrant's audience. Instead of covering a segment on length $\frac{T}{m^*}$, it serves a segment of length $\frac{T}{m^{**}} < \frac{T}{m^*}$. For the π_1^* corresponding to the case without quotas, audiences of both radios are such that:

$$F(\pi_1^*) = F\left(\pi_1^* + \frac{T}{m^*}\right) - F(\pi_1^*) > F\left(\pi_1 + \frac{T}{m^{**}}\right) - F(\pi_1)$$

the equilibrium constraint is thus breached, since radio 2 would increase its audience by relocating on $[0, \pi_1^* - \varepsilon]$.

The equilibrium with quotas thus features a cutoff π_1^{**} between the two radios such that:

$$F(\pi_1^{**}) = F\left(\pi_1 + \frac{T}{m^{**}}\right) - F(\pi_1) < F(\pi_1^*)$$

which imply $pi_1^{**} < \pi_1 *$ since F is increasing.

The total measure of genres broadcast at the equilibrium with quotas is thus $\pi_1^{**} + \frac{T}{m^{**}}$ with $pi_1^{**} < \pi_1 *$ and $\frac{T}{m^{**}} < \frac{T}{m^*}$. Therefore, $\pi_1^{**} + \frac{T}{m^{**}} < \pi_1 * + \frac{T}{m^*}$. □

Quotas with competing radios will thus have the same impact as with a monopoly radio: less popular genres will be evicted while more popular domestic titles will be repeated more often. The counter-productive effect of broadcasting quotas also exists when radios compete.

4 Extension: Profit-maximizing radios

This extension considers the case when the radios do not seek to maximize audience but profits coming from advertising revenues. Since the focus of this paper is on the impact of the quantity of advertising on programming choices, I assume that the price per listener charged to advertisers is constant (and equal to one). For the sake of simplicity, I also assume that the time devoted to ads is negligible and only consider the time devoted to music when assessing the effect of the constraint. Ads are broadcast to the whole audience (this rules out targeted advertising) which means their level is common for all listeners. The profits of a radio are thus the number of ads a times its audience.

The introduction of advertisements also affects consumers. While listening to the radio, a consumer also hears advertisement that represent a disutility. I assume this disutility is an increasing function a^r, $r > 1$, of the number a of advertisements. A consumer thus listen to the radio that provides him the highest utility, net of the disutility of advertisement and provided that the net utility is greater than zero.

The increased generality of the model comes at a cost in terms of tractability. In order to alleviate these issues, I use more specified (by still quite generic) forms of the utility and density functions. More specifically, I assume that the utility is a Dixit-Stiglitz one:

$$u(m_D, m_F, a) = (m_D^\rho + m_F^\rho)^{\frac{1}{\rho}} - a^r \tag{12}$$

with $\rho \in [0, 1]$ explicitly the substitutability between the two types of music and $r > 1$ denoting the concavity of the disutility of advertisements. For the density, let it follow an exponential distribution function of parameter λ, $f(\pi) = \lambda e^{-\lambda\pi}$ and of cumulative density function $F(\pi) = 1 - e^{-\lambda\pi}$. Here, λ provides a useful measure of the concentration of tastes.

4.1 Monopoly with advertisement

Let us first consider a radio operating as a monopoly. This case illustrates the core trade-off faced in the programming decisions, the sensitivity of this trade-off to the concentration of tastes and aversion to ads, and the reaction of the radio to different regulatory requirements

Absent any constraint on programming, a radio solves:

$$\max_{a, m_D(.), m_F(.)} \{F(b) - F(a)\} \tag{13}$$

$$\text{subject to: } \int_a^b [m_D(x) + m_F(x)]\,\mathrm{d}x \leqslant T \tag{14}$$

$$\forall x \in [a,b], u(m_D(x), m_F(x), a) \geqslant 0 \tag{15}$$

Since advertising revenues increase with the audience, the radio will always serve first the denser part of it, that is select the genres closest to zero. The essential trade-off in this case will be the choice between broadcasting new genres or using the same time endowment to increase programming for already-broadcast genres and increasing the level of advertising accordingly.

In order to explain the workings of this trade-off, first notice that a monopoly radio will always find optimal to saturate the time constraint (14) and consumers' participation constraint (15). For any given level of ads a, the latter thus defines the total amount of time that a monopoly radio must devote to any given genre in order to capture the corresponding listeners. Let $m(a)$ denote this quantity, with:

$$m(a) = 2^{-\frac{1-\rho}{\rho}} a^r \tag{16}$$

Making full use of the time endowment then means that the radio will broadcast $m(a)$ of all its selected genres, this broadcasting over a segment of length $T/m(a)$. The optimal level of advertisement a^* is then defined by the first-order condition:

$$F\left(\frac{T}{m(a^*)}\right) = \frac{T}{m(a^*)} \varepsilon_{(m(a^*),a^*)} f\left(\frac{T}{m(a^*)}\right) \tag{17}$$

where $\varepsilon_{(m(a),a)}$ is the elasticity of $m(a)$ with respect to a, which, in our case, is constant and equal to r.

This first order condition reads as follows: when the radio makes a marginal increase in the level of advertisement, it increases its profits by $F\left(\frac{T}{m(a^*)}\right)$, the measure of its audience. It must, however, compensate its listeners for the decrease in utility. For each genre, it must provide an additional quantity of music equal to $\varepsilon_{(m(a^*),a^*)}$. Compensating its whole audience thus costs $\frac{T}{m(a^*)}\varepsilon_{(m(a^*),a^*)}$ in terms of time endowment. This extra broadcasting time is obtained by cutting the programming on the less popular genres, which cater to a density of consumers $f\left(\frac{T}{m(a^*)}\right)$. The right-hand side of the condition is then the marginal loss of audience stemming from the necessity to compensate the other listeners for a higher level of advertisement.

This condition allows to underline the effects of the aversion to advertisement and of the concentration of tastes on programming.

Lemma 4.1. *The breadth of programming $\frac{T}{m(a^*)}$ is increasing with r and decreasing with λ. This means the breadth of programming is larger when consumers are more averse to advertisements* *and narrower when their distribution is more concentrated.*

Proof. The relationship between the optimal breadth of programming $\frac{T}{m(a^*)}$ and the characteristics of consumers is best seen when rewriting the first-order condition (17) as:

$$\begin{aligned}\frac{T}{m(a^*)}\varepsilon_{(m(a^*),a^*)} &= \frac{F\left(\frac{T}{m(a^*)}\right)}{f\left(\frac{T}{m(a^*)}\right)}\\ \Leftrightarrow \frac{T}{2^{\frac{\rho-1}{\rho}}a^r}r &= \frac{1}{\lambda}\left(\exp\left[\frac{T\lambda}{2^{\frac{\rho-1}{\rho}}a^r}\right]-1\right)\end{aligned} \tag{18}$$

Both the left- and the right-hand side of equation (18) are decreasing in a. If an interior solution exists however, the right-hand side is decreasing faster than the right-hand side and the problem is well-behaved. To see that, first notice that the second-order condition of the problem imposes[6]:

$$\frac{\lambda T}{m(a)} > \frac{r-1}{r} \tag{19}$$

Then, notice that the right-hand side decreases more steeply than the left-hand side if:

$$\begin{aligned}-\frac{r}{m(a)}\mathrm{e}^{\frac{\lambda T}{m(a)}} &< -\frac{r^2 T}{m(a)}\\ \Leftrightarrow \mathrm{e}^{\frac{\lambda T}{m(a)}} &> r\end{aligned}$$

Now, assume that this is not the case, that is $\mathrm{e}^{\frac{\lambda T}{m(a)}} \leqslant r$. Then,

$$\frac{\mathrm{e}^{\frac{\lambda T}{m(a)}}-1}{\lambda} \leqslant \frac{r-1}{\lambda}$$

Using the first-order condition to replace the left-hand side means it implies:

$$\begin{aligned}\frac{rT}{m(a)} &\leqslant \frac{r-1}{\lambda}\\ \Leftrightarrow \frac{\lambda T}{m(a)} &\leqslant \frac{r-1}{r}\end{aligned}$$

Which contradicts the second-order condition.

We thus know that the term $\frac{1}{\lambda}\left(\exp\left[\frac{T\lambda}{2^{\frac{\rho-1}{\rho}}a^r}\right]-1\right)$ which relates to the ration $F(T/m(a))/f(Tm(a))$ decreases more steeply with respect to a that the term $\frac{T}{2^{\frac{\rho-1}{\rho}}a^r}r$ which relates to consumers' elasticity to demand. Now, the condition for this to hold also with respect to r is:

$$-\frac{T}{m(a)}\left(1+(\mathrm{e}^{\frac{\lambda T}{m(a)}}-r)\log(a)\right) < 0$$

[6]See Appendix A.2.1 for a full derivation of this condition.

This is:

$$\mathrm{e}^{\frac{\lambda T}{m(a)}} - r > -\frac{1}{\log(a)}$$

which is implied by the condition $\mathrm{e}^{\frac{\lambda T}{m(a)}} > r$ demonstrated above.

Thus, when r increases, $F(T/m(a))/f(Tm(a))$ decreases more than $\frac{T}{2^{\frac{\rho-1}{\rho}}a^r}r$ for any given a. Since $F(T/m(a))/f(Tm(a))$ also decreases in a more than the other term, if $\tilde{r} > r$, then the equilibrium level of advertisement fulfilling the first-order condition must be lower for $\tilde{r}$ than for r. Because $m(a)$ is increasing in a, this means $\frac{T}{m(\tilde{a})} > \frac{T}{m(a)}$.

For the second part of the lemma, an increase in λ affects only the term $F(T/m(a))/f(Tm(a))$. F is unambiguously increasing in λ and f is decreasing in λ if $\lambda\frac{T}{m(a)} > 1$. Since $m(a)$ is the time devoted to any given genre, $\frac{T}{m(a)} > 1$, this is always the case. Thus, if a the optimal level associated with some value of λ then for any $\tilde{\lambda} > \lambda$,

$$\frac{T}{2^{\frac{\rho-1}{\rho}}a^r}r = \frac{1}{\lambda}\left(\exp\left[\frac{T\lambda}{2^{\frac{\rho-1}{\rho}}a^r}\right] - 1\right) < \frac{1}{\tilde{\lambda}}\left(\exp\left[\frac{T\tilde{\lambda}}{2^{\frac{\rho-1}{\rho}}a^r}\right] - 1\right)$$

The corresponding optimal level of advertisement $\tilde{a}$ needs to be larger than a in order to decrease the right-hand side term. This means $\tilde{a} > a$, $m(\tilde{a}) > m(a)$ and then $\frac{T}{m(\tilde{a})} < \frac{T}{m(a)}$. □

4.2 Broadcasting quota

Assume now that the regulator imposes to the monopoly commercial radio a broadcasting quota. More precisely, assume that the radio must devote a share at least $\alpha > 1/2$ of its time to domestic contents. The radio now solves the problem:

$$\max_{a, m_D(.), m_F(.)} \{F(b) - F(a)\} \tag{20}$$

$$\text{subject to: } \int_a^b [m_D(x) + m_F(x)]\,\mathrm{d}x \leqslant T \tag{21}$$

$$\forall x \in [a, b], u(m_D(x), m_F(x), a) \geqslant 0 \tag{22}$$

$$\int_a^b m_D(x)\mathrm{d}x \geqslant \alpha T \tag{23}$$

That is, program defined by equations (13) through (15) plus the additional constraint (23). As the quota constraint is global, that is, it applies to the whole programming, not to each genre, I first show that the radio optimally sets at each point a mix of domestic and foreign programming that exactly saturates the quota.

Lemma 4.2 (Monopoly quota programming)**.** *For any given level of advertisement a, the radio* *optimally selects a mix of domestic and foreign contents such that:*

$$\forall x \in [a,b], m_D(x) = \alpha\left(m_D(x) + m_F(x)\right) \tag{24}$$

Proof. The proof is identical to that of lemma 3.2 in the audience-maximizing case. □

The intuition of the proof is as follows. Let us assume that the radio sets a level of advertisement a and wants to cover a given segment $[0, \pi_\alpha]$. Because the cost in terms of time of capturing an interval $[\pi, \pi + \varepsilon]$ does not depend on π, $m_D(.)$ and $m_F(.)$ will be constant over the segment, and we can reason point-wise. This considerably simplifies to analysis of the consequences of a quota.

Proposition 4.1 (Monopoly quota)**.** *The breadth of programming* $\frac{T}{m_\alpha(a^*)}$ *under a quota* $\alpha > 1/2$ *of domestic contents is increasing in* α*. This means that imposing a quota of domestic contents increases the diversity of both domestic and foreign contents.*

Proof. Consumers' participation constraint implies that:

$$m_D = \left[1 + \left(\frac{1-\alpha}{\alpha}\right)^\rho\right]^{-\frac{1}{\rho}} a^r \tag{25}$$

Let us define β such that:

$$m = \left[\alpha^\rho + (1-\alpha)^\rho\right]^{-\frac{1}{\rho}} a^r = \beta a^r \tag{26}$$

By definition, β belongs to $\left[2^{\frac{\rho-1}{\rho}}, 1\right]$, is increasing in α and in ρ.

The first-order condition of the profit-maximization problem now writes:

$$\frac{rT}{\beta a^r} = \frac{\mathrm{e}^{\frac{\lambda T}{\beta a^r}} - 1}{\lambda} \tag{27}$$

and the result of lemma 4.1 remains valid, replacing the factor $2^{\frac{\rho-1}{\rho}}$ by β. Thus, the right-hand side of the condition decreases more steeply with a than the left-hand side. Moreover, $\frac{\mathrm{e}^{\frac{\lambda T}{\beta a^r}}-1}{\lambda}$ also decreases more steeply in α than $\frac{rT}{\beta a^r}$ does.

Let us then consider some $\alpha > 1/2$ and the associated optimal level of advertisement a. Define similarly $\tilde{\alpha}$, $\tilde{a}$ and $\tilde{\beta}$ the value of the parameter β for $\tilde{\alpha}$. Because of the relative slope of the two expressions, we know that:

$$\frac{rT}{\tilde{\beta} a^r} < \frac{\mathrm{e}^{\frac{\lambda T}{\tilde{\beta} a^r}} - 1}{\lambda} \tag{28}$$

Since the right-hand side is steeper in a than the right-hand side, $\tilde{a}$ must be lower than a. Hence, $m(\tilde{a}) < m(a)$ and $\frac{T}{m(\tilde{a})} > \frac{T}{m(a)}$. □

Proposition 4.1 shows that imposing a broadcasting quota increases both the share of domestic contents and the diversity of genres in the radio's programming. It however decreases the radio's profit.

4.3 Advertisement limitation

Assume now that the regulator imposes a limitation in the quantity of advertisement a radio can add to its programming (an ad cap). In this setup, a binding ad cap increases the diversity of programming and thus appears as a substitute to a cultural quota as far as diversity is concerned.

Proposition 4.2 (Monopoly ad cap)**.** *The breadth of programming increases when the ad cap $\overline{a}$ becomes more stringent, that is $\frac{T}{m(\overline{a})}$ is decreasing in $\overline{a}$.*

Proof. This proposition is fairly straightforward. If the quota $\overline{a}$ is biting, it means that $\overline{a} < a^*$, with a^* ad defined in lemma 4.1. Since the monopoly always saturate its consumers' participation constraint, it will program a quantity $m(\overline{a}) < m(a^*)$ along a breadth $\frac{T}{m(\overline{a})} > \frac{T}{m(a^*)}$. ☐

In terms of diversity, an ad cap is thus a substitute of a quota. This result contrast with that obtained by Richardson (2006) where an ad cap reduces the incentives for differentiation.

Th interesting point here is that these two measures are not commonly viewed as substitutes, rather as tools that are independent from each other. When, as here, they both impact programming and advertising level decisions, this assumption is not valid and the combination of the two can have major unwanted effects.

Proposition 4.3 (Monopoly quota and ad cap)**.** *When a monopoly radio is subject to a quota $\alpha > 1/2$ and an ad cap $\overline{a}$ such that the cap is binding, the breadth of its programming is narrower than without a quota. That is $\frac{T}{m_\alpha(\overline{a})}$ is decreasing in α.*

Proof. Consider a radio subject to an ad cap $\overline{a}$. It optimally broadcast $m(\overline{a})$ to all consumers between 0 and $\frac{T}{m(\overline{a})}$. Assume now that the regulator imposes a quota α such that the ad cap is still binding, that is, if a^*_α is the optimal level of advertisement with the quota disregarding the existence of the ad cap, $a^*_\alpha > \overline{a}$. In that case, the radio's profit is strictly increasing in a and it optimally selects the highest possible level of ads, that is $\overline{a}$. Because of the quota however, it must provide its listeners with a level of programming $m_\alpha(a) = \beta\overline{a} > 2^{\frac{\rho-1}{\rho}}\overline{}(a) = m(a)$ (the reasoning of 4.2 still applies: the quota is spread over all listeners). Because of the time constraint, it cannot provide this level of programming to all consumers between 0 and $T/m(a)$ and must restrict itself to the denser part of that segment, that is $\left[0, \frac{T}{m(\overline{a})}\right]$, with $\frac{T}{m(\overline{a})} < \frac{T}{m(a)}$.

Finally, since β is increasing in α, $m_\alpha(a)$ is also increasing in α and $\frac{T}{m(\overline{a})}$ is decreasing in α as long as the ad cap remains binding. ☐

The result of proposition 4.3 is similar to the results in section 3.1.2. Here, the assumption of an audience-maximizing radio with a fixed outside option for consumers is replaced by a binding ad cap that has the same effect: the radio is not willing to decrease its ad level and prefers to reduce its programming instead, sacrificing less popular genres in order to compensate listener of the most popular genres for the suboptimal mix. If both the ad cap and the quota are very stringent, it is even possible that the breadth of programming under the two constraint is narrower than that would prevail if they were both absent. This is made more likely if the two types are less substitutable (low ρ) since the radio must provide a higher level of contents in order to compensate listener for a less-than-optimal mix between types.

5 Conclusion

In this paper, I set out to show how the trade-off between catering to the denser part of the audience and conquering listeners further away from the most popular genres entailed a trade-off between repetition (of popular songs) and diversity. I showed how, with consumers liking only one genre and with complementarity between domestic and foreign music, that trade-off made broadcasting quotas of domestic content counter-productive in terms of diversity. That effect is heightened in a competitive setup, which also features its own diversity-reducing property. These results hint at the idea that using market shares as a proxy for diversity may be only part of the story, and that comprehensive data on genres are needed to assess the impact of globalization on programming.

The introduction of profit-maximizing motives (rather than pure audience-maximizing actors) suggests that this effect may be mitigated or even reversed if the disutility to advertisement is strong relative to the loss of utility due to a suboptimal mix. However, the trade-off reappears when the radios are also subject to an ad cap, which is by and large the case when quotas exist. This result suggests that ad caps and quotas should be seen as substitutes and in general not used in combination.

Because the precise delimitation of what “genre”, “domestic” and “foreign” mean is not specific to radio broadcasting, the same reasoning applies to various setups of the media industries. The choice between more songs of a popular genre or some of a less-popular one is akin to that between a new season or a ripoff of an established series and making a completely new one, without an existing fan base. Thus, quotas setting a floor or a cap of any given kind of programming are bound to produce the same kind of effect, e.g. restriction on the days feature films can be shown on TV should lead to the production of television movies that mimic those films and a reduction of the share of more confidential movies broadcast.

Further work should extend the analysis of profit-maximizing radios to the competition framework. Preliminary work suggests that the results stem naturally from the insights of audience-maximizing competition and of the monopoly with advertisement.

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A Appendix

A.1 Audience-maximizing radios

A.1.1 Monopoly quota programming

The radio audience maximization program is:

$$\max_{(a,b,m_F(.),m_D(.))} \{F(b) - F(a)\} \tag{29}$$

$$\int_a^b m_F(\pi)\mathrm{d}\pi \leqslant (1-\alpha)T \tag{30}$$

$$\int_a^b m_D(\pi)\mathrm{d}\pi \leqslant \alpha T \tag{31}$$

$$\forall \pi \in [a,b], u(m_D(\pi), m_F(\pi)) \geqslant \gamma \tag{32}$$

From the case without quota, we know that $a = 0$, and than b will be the optimal π^{**}

Since we assume the quota to be a real constraint, we know that (30) is binding. Since at any level, marginal utility is positive, the monopoly radio can always increase its audience by broadcasting more music, hence (31) is also biting, and finally, audience maximization means that (32) is biting.

Firstly, let $m_D(m_F)$ denote:

$$m_D(m_F) = \arg\min_{m_D} \{m_D\}$$
$$\text{s.t. } u(m_D, m_F) \geqslant \gamma$$

From what we have seen in construction the optimal, non-quota, programming, we kinow that $m_D(m_F)$ is a well-defined, monotonously decreasing mapping. Secondly, since F is monotonously increasing in π, maximizing $F(\pi)$ is equivalent to maximizing π itself. This allows to reduce the above problem to a standart minimum-time optimal control problem, where π is the target, $m_D(m_F)$ links the two controls $m_F(.), m_D(.)$ and the constraints (30) and (31) provide the evolution and the transversality conditions of the problem.

I rewrite the problem as a canonical maximum-time problem :

$$\max_{m_F(.)} \left\{ \int_0^\pi 1\mathrm{d}u \right\}$$
$$\dot{q}(u) = m_F(u), \quad \dot{t}(u) = m_D(m_F(u))$$
$$q(0) = t(0) = 0, \quad q(\pi) = (1-\alpha)T, \quad t(\pi) = T$$

The Hamiltonian associated with this maximum-time problem is :

$$\mathcal{H} = p_0 + p_1(u)m_F(u) + p_2(u)m_D(m_F(u))$$

The necessary conditions are :

$$\frac{\partial \mathcal{H}}{\partial q} = \frac{\partial p_1}{\partial u} = 0 \tag{33}$$

$$\frac{\partial \mathcal{H}}{\partial t} = \frac{\partial p_2}{\partial u} = 0 \tag{34}$$

$$\frac{\partial \mathcal{H}}{\partial m_F} = p_1 + p_2\frac{\partial m_D}{\partial m_F} = 0 \tag{35}$$

Equations (33) and (34) tell us that p_1 and p_2 are constants. Equation (35) means that at the optimum there is a affine relation between m_F and m_D, that is :

$$m_D = -\frac{p_1}{p_2}m_F + k_1 \tag{36}$$

Now, consider the transversality conditions. At the optimum, the second condition is fulfilled when the radio uses up its whole quota Q. Since we assumed that the quota has some bite, it follows that the radio is always willing to do so, since it allows it to provide more utility to its listeners. For the same reason, the radio is always willing to use its whole time endowment T. Then, the second transversality condition rewrites as:

$$k_1\pi - \frac{p1}{p2}\int_0^{\pi} m_F(u)\mathrm{d}u = \alpha T \tag{37}$$

which allows us to simplify the expression of $m_D(m_F)$ as;

$$m_D = \frac{\alpha}{1-\alpha}m_F + k_1 \tag{38}$$

Since we want both $m_D(x)$ and $m_F(x)$ to be nil for any $x > \pi^{**}$ it is necessary that $k_1 = 0$. Hence, for all $u \in [0, \pi^{**}]$,

$$m_D(x) = m_F(x)\frac{\alpha}{1-\alpha} \tag{39}$$

Equation 39 defines a function $\widetilde{m}_D(m_F)$ strictly increasing in m_F. Since $m_D(m_F)$ defined above is a decreasing function, the optimal programming level of foreign music m_F^{**} is the one that simultaneously satisfies (32) and (39), which is unique. Because both conditions do not depend on x, that optimum m_F^{**} does not depend on x. According constraint (32), this value is given by:

$$u\left(\frac{\alpha}{1-\alpha}m_F^{**}, m_F^{**}\right) = \gamma \tag{40}$$

or equivalently, m_D^{**} such that:

$$u\left(m_D^{**}, \frac{1-\alpha}{\alpha} m_D^{**}\right) = \gamma \tag{41}$$

A.1.2 No pure simultaneous-move equilibrium

This subsection shows that the simultaneous-move competition game admits no Nash equilibrium. The outline of this demonstration is as follows: in a first part, I show that the strategy space can be reduced to the choice of the segment of genres covered. I then show that there exists no symmetric equilibrium and then that asymmetric equilibria cannot exist either.

Lemma A.1. *The strategy of a radio can be fully expressed by the segment $[\pi_i, \Pi_i]$ covered by its programming.*

Proof. The strategy of radio i is defined by its programming mix (m_d^i, m_F^i) and the segment of genres it covers $[\pi_i, \Pi_i]$. The monopoly case shows that the strict concavity of u entails that the couple (m_d^i, m_F^i) maximizing utility on a given $[\pi_i, \Pi_i]$ is unique.

It is straightforward that strategies using a different mix are strictly dominated: if a radio chooses a different mix, the other one can offer to the segment of listeners the same level of utility with less total programming, and still have some spare time to capture listeners outside of the other radio's audience. Such strategies will thus never be part of an equilibrium neither be a credible threat. □

A strategy is given by (π_i, Π_i). In what follows, results are clearer when one bears in mind that the lower the size $\Pi_i - \pi_i$ of the segment, the higher the utility of agents in that segment. Using that property, it is possible to delineate some characteristic of a potential equilibrium. Since the market can never be totally covered, the equilibrium profits are positive. In any case, a radio can move to the free tail of the types' distribution and make some profit there. Moreover, I argue there exists no symmetric, pure-strategies equilibrium, but there can be an asymmetric equilibrium. To assert that, I first show that a candidate equilibrium cannot be symmetric, that the program schedules must have disjoint support and that consumer surplus must be equal to zero. This allows me to show that if consumer density f is strictly decreasing, there is no pure-strategies, simultaneous-move equilibrium.

Lemma A.2 (No symmetric equilibrium)**.** *The competition game admits no pure-strategy, symmetric equilibrium.*

Proof. Assume there exists a pure-strategies, symmetric equilibrium E with $\pi_i = \pi_j = \pi$ and $\Pi_i = \Pi_j = \Pi$. In such case, radios share half the audience on this segment. However, radio i can reduce its Π_i by a small ε. It would then provide a strictly higher utility to listeners over the $[\pi, \Pi - \varepsilon]$ segment, thereby capturing all the audience. Such a deviation is profitable when $F(\Pi - \varepsilon) - F(\pi) > \frac{1}{2}(F(\Pi) - F(\pi))$, which is true for some ε on any non-degenerate segment $[\pi, \Pi]$. A symmetric situation thus cannot be an equilibrium. □

Thus, if an equilibrium exists, it is asymmetric in at least one of the choice parameters.

Lemma A.3 (Disjoint support). *If an equilibrium exists, the two programs have disjoint support, that is either* $\Pi_i \geqslant \pi_j$ *or* $\Pi_j \geqslant \pi_i$.

Proof. First, notice that if at equilibrium there is an overlap between the two supports, then the two radios provide the same consumer surplus on the overlap (and hence on all the covered subsection of the market). If it were not the case, the radio with the lower consumer surplus gets no audience from the overlap and finds profitable to serve an uncovered part of the audience.

Next, since both radios offer the same surplus over the overlap, each captures half the audience there. This can be an equilibrium strategy only if the radio "on the left" (*i.e.* serving the denser part of consumers) is already serving all the market between 0 and the overlap region. Otherwise, abandoning the overlap region to serve an audience closer to 0 (and hence more numerous) would always be profitable. Then, an equilibrium with an overlap will always have the form $[0, \Pi_i], [\pi_j, \Pi_j]$ with $\Pi_i \geqslant \pi_j$ (that is an overlap on $[\pi_j, \Pi_i]$).

Since on the two radios provide the same utility to their listeners (see first subparagraph of this proof), it is then profitable for radio j to relocate on $[0, \Pi_j - pi_j - \varepsilon]$ and capture all demand: up to the small ε, the width of the segment $[0, \Pi_j - pi_j - \varepsilon]$ is equal to that of the segment $[\pi_j, \Pi_j]$ but located on a denser part of the market, which means a larger audience.

Thus, an equilibrium cannot feature an overlap in programs' supports. □

Lemma A.4 (Zero consumer surplus). *If an equilibrium exists, then the consumer surplus of the listeners of both radios is equal to zero.*

Proof. From lemmas A.2 and A.3, we know that an equilibrium is of the form : $([0, \Pi_i], [\Pi_i, \Pi_j])$. Let $s_l = u(mD^l, m_F \mathrm{l}) - \gamma$ be the net surplus for a consumer listening to radio $l \in \{i, j\}$. Obviously, s_i and s_j are positive, else the radios would have no listeners.

If $s_j > 0$, radio j's listeners enjoy a positive surplus. Radio j can then reduce its level of programming until $s_j = 0$ in order to serve a positive measure of consumers located between Π_j and $\Pi_j + \varepsilon$, thus increasing its advertising revenues. Hence, $s_j = 0$.

For the same reasons, if $s_i > s_j$, radio i can cut in its programming in order to capture consumers between Π_i and $\Pi_i + \nu$, encroaching on radio j's public. Thus, at equilibrium, $s_i \leqslant s_j$.

Putting all conditions together gives: $0 \leqslant s_i \leqslant s_j = 0$. □

The last two steps are to show that equilibrium, both profits must be equal and that consumer density f strictly decreasing implies that radio j's profit are always lower than radio i's under the other equilibrium conditions.

Proposition A.1 (No simultaneous equilibrium). *The symmetric simultaneous-move competition game admits no Nash equilibrium.*

Proof. Let A_i denote radio i' audience at a candidate equilibrium and A_j radio j's. If $A_i > A_j$, radio j can profitably take radio i's customer with a slightly narrower support of genres and increase its profit. Hence at equilibrium, $A_i = A_j$.

From the preceding lemmas, we know that at a candidate equilibrium both radios broadcast the same level m^* of music of each genre they cover, with $m^* = m_D^* + m_F^*$ such that (m_D^*, m_F^*) is the more efficient mix to reach $u(m_D^*, m_F^*) = \gamma$. In that case, the two radios have the following audiences:

$$A_i = \left\{ F\left(\frac{T}{m^*}\right) \right\}$$
$$A_j = \left\{ \left[F\left(\frac{2T}{m^*}\right) - F\left(\frac{T}{m^*}\right) \right] \right\}$$

Because f is strictly decreasing and the lengths of the intervals covered are the same, the density of consumers on $\left[0, \frac{T}{m^*}\right]$ is greater than on $\left[\frac{T}{m^*}, \frac{2T}{m^*}\right]$. Then, $V_i > V_j$, which is not compatible with an equilibrium since radio j will want to take radio i's location. $\square$

Since no such equilibrium exists, one would be tempted to see what happens when mixed strategies are allowed. However, the strategy space (the set of closed intervals $[a, b]$ of $\mathbb{R}_+$ such that $b - a \leqslant T/m^*$) is cumbersome and the actual meaning of mixed programming strategies not clear. For that reason, I prefer to consider a sequential game where one radio acts as a Stackelberg leader, choosing its location first.

A.1.3 Strategy selection

In order to show that the "popular incumbent" strategy is a dominant strategy for the incumbent, I need to show that both strategies may lead to equilibrium candidates and then show the popular one is preferred by the incumbent.

Best responses and outcomes In what follows, I show that the popular strategy always leads to a single equilibrium candidate, while allowing entry on the most popular genres also leads to an equilibrium candidate for any distribution of probability.

Popular incumbent The incumbent broadcasts on a $[0, \pi_1]$ segment, and sets a surplus s_1 such that radio 2 is better off by capturing monopoly profits on a $[b_2, c_2], b_2 > \pi_1$ segment.

Lemma A.5. *For any decreasing distribution, there exists a unique "popular" strategy that makes radio 2 indifferent between entering on less popular genres and competing for more popular ones. Programs are then on the segments* $[0, \pi_1^l], [\pi_1^l, \pi_1^l + \frac{T}{m^*}]$.

Proof. The popular strategy means that radio 1 locates on a $[0, x]$ segment. Radio 2 best response is then of of the two:

1. Locating at $\left[x, x+\frac{T}{m^*}\right]$, thus making monopoly profit on that (less popular) part of the audience.

2. Competing with 1 for the more popular genres, locating on $[0, x-\varepsilon]$, thus offering a slightly larger utility to listeners.

Since the second response entails (near)-zero profit for radio 1, it selects the greatest x such that radio 2 chooses the first response, that is:

$$\max_x \{F(x)\}$$
$$\text{s.t. } F(x) \leqslant F\left(x+\frac{T}{m^*}\right) - F(x)$$

Now, I want to show that the maximum argument x^* of this program exists and in unique. Firstly, notice that

$$\frac{\partial}{\partial x}\left[F\left(x+\frac{T}{m^*}\right) - F(x)\right] = f\left(x+\frac{T}{m^*}\right) - f(x)$$

Since f is monotonously decreasing and m^* does not depend on the location, $f\left(x+\frac{T}{m^*}\right) < f(x)$, which means $F\left(x+\frac{T}{m^*}\right) - F(x)$ is also monotonously decreasing in x. For $x=0$, it is trivial that $F\left(\frac{T}{m^*}\right) > F(0) = 0$, and because of f decreasing, it is straightforward that

$$F\left(\frac{T}{m^*}\right) > F\left(\frac{2T}{m^*}\right) - F\left(\frac{T}{m^*}\right)$$

The intermediate-values theorem then implies that there exists one unique x such that $F(x) = F\left(x+\frac{T}{m^*}\right) - F(x)$ and that it is the greatest x such that $F(x) \leqslant F\left(x+\frac{T}{m^*}\right) - F(x)$. □

Niche incumbent This strategy is almost the mirror image of the previous one. Here, the incumbent uses consumer surplus in order to "squeeze" the entrant on the left of the demand. The entrant thus captures the most popular titles, but is constrained on its right by the incumbent.

Lemma A.6. *For any decreasing distribution, there exists a unique strategy $[y^*, z^*]$ that maximizes Radio 1's audience and has Radio 2 preferring to serve the $[0, y^*]$ segment to competing for Radio 1's listeners.*

Proof. A niche strategy is a segment $[y, z] \subset \mathbb{R}_+$ chosen as location by Radio 1. Radio 2's response can then be:

1. Settle on $[0, y]$ for an audience $F(y)$

2. Serve all listeners on $[0, y]$ and compete for some on $[y, z]$.

In order to fully characterize the second option, let u_1 denote the utility of consumers listening to Radio 1, $\tilde{m}(s) = (\tilde{m}_D(s), \tilde{m}_F(s))$ the optimal mix corresponding to a segment of length s, and x_2:

$$\begin{aligned} x_2 =& \arg\max_{x_j} \{F(x_j^r)\} \\ \text{s. t.: } & u\left(\tilde{m}\left(\frac{T}{x_j}\right)\right) \geqslant u_1 \end{aligned}$$

that is the larger audience that Radio 2 can get while providing a utility as least equal to that provided by Radio 1.

In order to maximize its audience, Radio 1 must then ensure that $F(y) \geqslant F(x_2)$. Notice that this also implies that $F(y) \geqslant F(z) - F(y)$ (since f is decreasing). Radio 1 problem is thus:

$$\begin{aligned} &\max_{y<z} \{F(z) - F(y)\} \\ &\text{s. t.: } F(y) \geqslant F(x_2) \end{aligned}$$

It is clear that for $y = \frac{T}{m^*}$, the constraint is fulfilled. The set of optimal strategies with entry on the most popular titles is thus non-empty and admits a larger element in terms of audience for Radio 1. □

Now, let us compare the payoffs of the optimal strategies of each kind. I show that those of the niche strategy are always larger than those of the other strategy.

Let l refer to the popular strategy and r denote niche strategy. Let A_k^h denote the audience of radio k in equilibrium candidate $h \in \{l, r\}$. Let $[0, x]$ denote the optimum location of Radio 1 with the popular strategy and $[0, y], [y, z]$ denote the optimum location of Radios 2 and 1 respectively in the other case.

From the first conditions for best-response from Radio 2, we know that Radio 2 must has an audience at least as large as Radio 1 in either case, that is:

$$A_2^l \geqslant A_1^l \qquad \text{and} \qquad A_2^r \geqslant A_1^r$$

Assume now that Radio 1 strictly prefers the niche strategy, that is $A_1^r > A_1^l$. This implies:

$$\begin{aligned} &A_2^r \geqslant A_1^r > A_1^l \\ &A_2^r > A_1^l \Leftrightarrow F(y) > F(x) \\ &\qquad\quad \Leftrightarrow y > x \text{ since } F \text{ is strictly increasing} \end{aligned}$$

Thus,

$$
\begin{aligned}
A_1^r &= F(z) - F(y) \\
&\leqslant F\left(\frac{T}{m^*} + y\right) - F(y) \text{ since } z - y < \frac{T}{m^*} \\
&\leqslant F\left(\frac{T}{m^*} + x\right) - F(x) \text{ since } x < y \text{, move to a denser part of the audience} \\
&\leqslant A_2^l \\
A_1^r &\leqslant A_1^l \text{ since } A_1^l = A_2^l
\end{aligned}
$$

Thus, $A_1^r > A_1^l \Rightarrow A_1^r \leqslant A_1^l$, which is obviously contradictory. Thus, $A_1^l \geqslant A_1^r$, which means that radio 1 always weakly prefers the entry on the less popular genres.

A.2 Profit-maximizing radios

A.2.1 Second-order condition

The second-order condition of the monopoly radio profit-maximization problem is:

$$
-\frac{T}{m(a)}\frac{\lambda r}{a^{r+1}}\mathrm{e}^{\frac{\lambda T}{m(a)}}\left(2^{\frac{1-\rho}{\rho}}rT\lambda - a^r(r-1)\right) < 0
$$

which is equivalent to:

$$
\begin{aligned}
&2^{\frac{1-\rho}{\rho}}rT\lambda - a^r(r-1) > 0 \\
\Leftrightarrow &\frac{\lambda T}{m(a)} > \frac{r-1}{r}
\end{aligned}
$$